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Research article

Application of diameter distribution model for volume estimation in *Tectona grandis* L.f. stands in the Oluwa forest reserve, Nigeria

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Abstract: The ability to predict the distribution of diameters in a stand is essential for forest managers to make informed management decisions such as prescription of silvicultural treatments and harvesting regimes. Such information is preferably derived from suitable distribution model. This study evaluated the performance of four distribution models in describing the structure of the teak stands in Oluwa Forest Reserve, Nigeria. Data were collected from 12 temporary sample plots of 20 × 20 m size in the teak stand. Maximum likelihood estimator was used to fit the distribution models: beta, gamma, Johnson SB, and Weibull to the diameter data from the teak stand. Relative rank-sum derived from four indices was used to conclude on the most suitable distribution for the stand. The results showed that the Weibull distribution was the most suitable function for the teak stand with a relative rank-sum of 4.0. Application of Weibull distribution together with suitable height-diameter and volume models estimated yield of 136.281 m^3 ha^{-1} within timber size class (diameter ≥30 cm). And a total of 309.640 m^3 ha^{-1} was estimated for the stand. Other product specifications were also provided. This would help in the routine management of the stand.
Keywords: Johnson SB - Stand structure - Teak - Weibull distribution - Yield.

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INTRODUCTION

Tree diameter distribution is used to determine the structure of a forest stand (Rouvinen & Kuuluvainen 2005). The ability to predict the distribution of diameters in a stand help forest manager to make informed decisions such as prescription of silvicultural treatments and determination of distribution (Carretero & Álvarez 2013). Knowledge of diameter distributions forms the basis for deciding when a stand can be economically harvested for a specific product (Ekpa *et al.* 2014). Diameter structure is an important stand characteristic on which we can evaluate the stability, growth, volume production, structure of assortment, and maturity (Gorgoso-Varela & Rojo-Alboreca 2014).

Stand yields have also been predicted based on the assumption that diameter distribution of a stand can be characterized by a probability density function (Poudel & Cao 2013). In forestry, various distribution functions such as normal (Nanang 1998), gamma (Nelson 1964, Gorgoso-Varela *et al.* 2020), lognormal (Mirzaei *et al.* 2016), Johnson's SB (Özçelik *et al*. 2016, Ogana & Ekpa 2020), beta (Mayrinck *et al.* 2018, Gorgoso-Varela *et al.* 2020), and Weibull (Sun *et al.* 2019, Schmidt *et al.* 2020, Gorgoso-Varela *et al.* 2020), logit-logistic (Wang & Rennolls 2005, Ogana *et al.* 2018) have been successfully used to describe tree diameter distributions of forest stands. The application of distribution to quantify stand volume in production forests has been relatively few in Nigeria (*e.g.* Ajayi 2013, Ogana & Ekpa 2020). Quantifying yield by size classes is harbinger for prescription of effective silvicultural treatment and harvesting regime for any forest stand.

The *Tectona grandis* L.f. is a fast-growing tropical hardwood tree species belonging to the family of Verbenaceae. Teak is one of the most widely cultivated exotic species in Nigeria because of its good anatomical and physical properties (Miranda *et al.* 2011). It is also multipurpose tree species and as such, there is continuous demand for its products (Miranda *et al.* 2011). The teak plantation in Oluwa Forest Reserve is a production forest established for pole and timber. There is dearth of information on product yield for the teak

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plantation. This information is necessary for the management of the plantation. Therefore, the main purpose of the study was to apply a suitable diameter distribution function to quantify the stand volume by size classes for effective management of the stand.

METHODOLOGY

Study area and data

The data used in this study were from the teak plantation (about 3 ha size) in the Oluwa Forest Reserve (FR) of Southwestern Nigeria. The Oluwa Forest Reserve is situated between 6º 55′ and 7º 20′ N and longitude 3º 45′ and 4º 32′ E, and occupies an area of 87,816 ha (Ogana & Ekpa 2020). Oluwa has an annual rainfall in the range 1700 to 2200 mm, an average annual temperature of 26°C, and a mean elevation of 123 m above sea level (Onyekwelu *et al.* 2006). Establishment of large-scale plantations in the reserve started in early 1960s. *Gmelina arborea* Roxb. and *Tectona grandis* L.f. are the dominant plantation species in Oluwa FR.

Data were collected from twelve sample plots of 20 m × 20 m in size in the teak stand. Diameter measurements of all trees (over bark) at breast height (1.3 m above ground, DBH) were measured with diameter tape to an accuracy of 0.1 cm. Their corresponding height (H) measurements were also taken with hypsometer. The measured variables were used to calculate the basal area (BA, m^2) and tree volume (V, m^3). The descriptive statistics of the data set is presented in table 1.

Table 1. Descriptive statistics of the teak data from Oluwa forest reserve.

Statistics	DBH (cm)	H (m)	BA (m^2)	V (m^3)
Mean	24.1	16.1	0.051	0.375
Standard Deviation	8.4	3.8	0.037	0.261
Minimum	10.0	8.4	0.008	0.038
Maximum	54.0	29.5	0.229	1.410
Kurtosis	0.55	0.26	4.384	1.091
Skewness	0.74	0.58	1.747	1.156
N = 379 trees				

Distribution models

The plot of skewness against kurtosis was first used to narrow the search for a probability density functions (pdf) that described the diameter distribution of the teak data (Fig. 1). The graph showed that the observed point was within the beta and close to Weibull and gamma distributions. Thus, these functions were used to fit the diameter data. Johnson SB function was also evaluated because studies (*e.g.* Wang & Rennolls 2005) has shown that it occupies the same region in the skewness and kurtosis plane as the beta distribution.

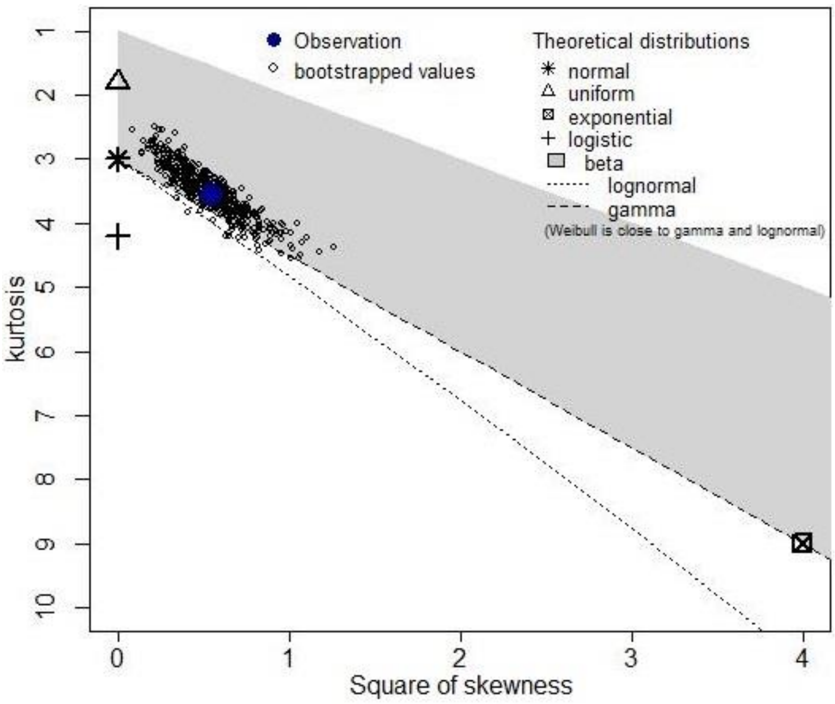

Figure 1. Skewness and kurtosis plane showing the plots of selected distributions.

i. *Beta distribution function*:

$$f(x) = \frac{1}{(b-a)B(\alpha,\beta)}\left(\frac{x-a}{b-a}\right)^{\alpha-1}\left(1-\frac{x-a}{b-a}\right)^{\beta-1} \quad [1]$$

$$B(\alpha,\beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} \quad [2]$$

where, $f(x)$ is the pdf; α, β represent the shape parameters; a, b are the location and scale parameters, respectively; Γ is the gamma function. $a \leq x \leq b$, $\alpha > 0$, $\beta > 0$.

ii. *Gamma distribution function*:

$$f(x) = \frac{(x-\gamma)^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} \cdot e^{\left[-\left(\frac{x-\gamma}{\beta}\right)\right]} \quad [3]$$

where, α is the shape parameter, β represents an inverse scale parameter, γ is the location parameter ($\gamma = 0$ for the two-parameter gamma distribution) and $\Gamma(.)$ represents the gamma function, with $x > \gamma$, $\alpha > 0$ and $\beta > 0$.

iii. *Johnson S_B distribution* (Johnson 1949):

$$f(x) = \frac{\delta}{\sqrt{2\pi}} \cdot \frac{\lambda}{(\xi+\lambda-x)(x-\xi)} \cdot e^{-\frac{1}{2}\left[\gamma+\delta\cdot ln\left(\frac{x-\xi}{\xi+\lambda-x}\right)\right]^2} \quad [4]$$

where, ξ and λ represent the location and scale parameters, respectively; γ and δ are the shape parameters; $\xi < x < \xi + \lambda$, $-\infty < \xi < +\infty$, $-\infty < \gamma < +\infty$, $\lambda > 0$, and $\delta > 0$.

iv. *Weibull distribution* (Weibull 1951):

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} e\left[-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right] \quad [5]$$

where, α is the shape parameter ($\alpha > 0$); β represents the scale parameter ($\beta > 0$); γ is the location parameter.

Fitting method and evaluation statistics

The method of maximum likelihood estimation was used to fit the functions to the teak data through the optim function in R (R Core Team 2017). Evaluation was based on Kolmogorov-Smirnov (Dn), Anderson-Darling (AD), Cramer-von Mises (W^2) and Bayesian information criterion (BIC). The smaller the indices, the better the distribution fit the data set. Furthermore, relative rank sum proposed by Poudel & Cao (2013) was applied to decide on the best distribution for the teak stand. It is expressed:

$$R_i = 1 + \frac{(m-1)(S_i - S_{min})}{S_{max} - S_{min}} \quad [6]$$

where, R_i represents the relative rank of distribution i (i = 1, 2, …, m); m is the number of distribution assessed (4 distributions), S_i is the fit index value of distribution i; S_{max} and S_{min}, represent respectively the maximum and minimum values of S_i. Relative rank is a real number between 1 (best) and 4 (worst). For each distribution, the relative ranks were summed for the four evaluation statistics. Distribution with the smallest relative rank sum was selected as the candidate distribution for the teak stand.

Application of the distribution

Table 2. Height-diameter models and tree volume equations for the teak stand in Oluwa forest reserve.

Model name	Form	Reference	Eq.
H-D models			
Chapman-Richards	$h = 1.3 + b_0(1-e^{-b_1 d})^{b_2}$	Richards (1959)	[7]
Korf	$h = 1.3 + b_0 e^{(-b_1 d^{-b_2})}$	Lundqvist (1957)	[8]
Logistic	$h = 1.3 + \frac{b_0}{1+b_1 e^{-b_2 d}}$	Pearl & Reed (1920)	[9]
Naslund	$h = 1.3 + \frac{d^2}{(b_0+b_1 d)^2}$	Näslund (1936)	[10]
Weibull	$h = 1.3 + b_0\left(1-e^{b_1 d^{b_2}}\right)$	Yang *et al.* (1978)	[11]
Volume Equations			
Combined variable	$v = b_0 + b_1 d^2 h$	Laar & Akçar (2007)	[12]
Constant form factor	$v = b_1 d^2 h$	Laar & Akçar (2007)	[13]
Logarithmic	$v = b_1 d^{b_2} h^{b_3}$	Laar & Akçar (2007)	[14]
Generalized logarithmic	$v = b_0 + b_1 d^{b_2} h^{b_3}$	Laar & Akçar (2007)	[15]

Note: b_0, b_1, b_2 = model parameters; h = total tree height (m); d = diameter at breast height (cm); v = volume (m^3) e = base of the natural logarithm.

The best distribution based on the evaluation statistics was applied to estimate the volume per hectare for specified diameter classes. To do this, suitable height-diameter (H-D) and volume models for the teak stand were first identified; so that the mean height for a given diameter class can be estimated. The diameter class midpoint and estimated mean height were inserted in the volume equation to derive the class mean volume. The product of the mean volume and the density (trees ha^{-1}) produced the volume per ha of that class. Since there are no suitable H-D and volume models for the teak stand in Oluwa FR, the candidate models in table 2 were evaluated.

The models were assessed based on mean absolute bias (MAB), root mean square error (RMSE) and BIC. The smaller the values of the indices, the better the model. Relative rank was also used to decide on the best H-D and volume models for the teak stand.

RESULTS

Fits of the distribution models

The estimated parameters of the beta, gamma, Johnson SB and Weibull distributions and their corresponding evaluation statistics are presented in table 3. The results showed that the Weibull distribution had the smallest Kolmogorov-Smirnov (Dn), Anderson-Darling (AD), Cramer-von Mises (W^2) and Bayesian information criterion (BIC) of 0.0309, 0.2649, 0.0378, and 2773, respectively with a least relative rank sum of 4.00. This was followed by beta and Johnson SB distributions with relative rank sum of 7.50 and 11.42, respectively. The gamma distribution had the largest relative rank sum (15.67), as such, ranked last. Thus, based on the evaluation statistics the Weibull was selected as the most suitable distribution for the teak stands.

The graph of the observed and predicted diameter distributions from beta, gamma, Johnson SB and Weibull of the teak stand is presented in figure 2. The predicted distributions were reasonable and compared well with the observed. The diameter distribution of the teak stand was slightly skewed to the right (*i.e.*, positive skewness).

Table 3. Estimated parameters and the evaluation statistics of the four distributions.

Distributions	Parameters				Evaluation statistics				
	location	scale	shape1	shape2	Dn	AD	W^2	BIC	∑R
Beta	9.1639	75.323	2.2061	7.5754	0.0319 (1.35)	0.3168 (1.42)	0.0448 (1.40)	2780 (3.33)	7.50
Gamma		0.3556	8.5675		0.0394 (4.00)	0.6354 (4.00)	0.0908 (4.00)	2781 (3.67)	15.67
Johnson SB	7.1264	62.3499	1.488	1.3566	0.0359 (2.76)	0.4206 (2.26)	0.0624 (2.39)	2782 (4.00)	11.42
Weibull	9.4408	16.4736	1.8083		0.0309 (1.00)	0.2649 (1.00)	0.0378 (1.00)	2773 (1.00)	4.00*

Note: Values in parenthesis are the relative ranks; ∑R: relative rank sum; * means selected distributions.

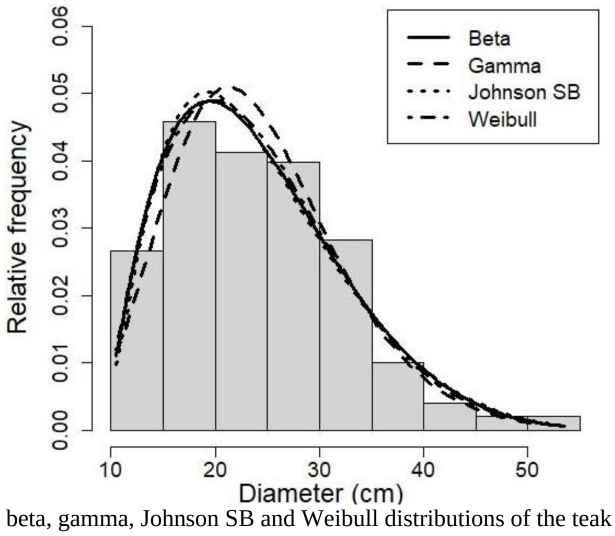

Figure 2. Observed and fitted beta, gamma, Johnson SB and Weibull distributions of the teak stand.

Height-diameter and volume models and application

The results of the fitted H-D models showed that Logistic model was the best function for estimating tree height in the teak stand (Table 4). It had the smallest MAB, RMSE and BIC of 2.175, 2.892, 1990 with a relative rank sum of 3.24. The Weibull and Chapman-Richards H-D functions also performed well with relative rank sum of 6.60 and 7.27, respectively. In fact, the Weibull model, had the smallest mean bias. The Korf model had the largest relative rank sum of 13.59. The five H-D models produced tree height trajectories that are consistent with biological realism: monotonic increment, inflection point and asymptote (Fig. 3).

In the case of the fitted volume functions, the generalised Logarithmic had the smallest MAB and RMSE of 0.046 and 0.068, respectively with the least relative rank sum of 3.01. The indices of Logarithmic volume model were approximately like those of the generalised Logarithmic model. It had relative rank sum of 3.23. Constant form factor model had the largest MAB, RMSE and BIC; as such ranked last. Though, the generalised Logarithmic had the smallest relative rank sum, it was not selected as the candidate volume model for the teak stand because one of its parameters was not significant ($p > 0.05$). The residual plots of the selected height-diameter (Logistic H-D) and volume (Logarithmic) models are presented in figure 4a and b. Both residual plots showed a relative horizontal band within the range of ±4 m and ±0.4 m^3 for the Logistic H-D and Logarithmic volume models, respectively.

Table 4. Parameters and fit indices of the height prediction models for the teak data

Models	**Parameters**				**Evaluation statistics**			
	b_0	**b_1**	**b_2**	**b_3**	**MAB**	**RMSE**	**BIC**	**∑R**
H-D models								
Chapman-Richards	20.020	0.070	1.248		2.176	2.901	1993	7.27
					(1.47)	(2.80)	(3.00)	
Korf	25.200	9.546ns	0.932		2.185	2.912	1996	13.59
					(3.59)	(5.00)	(5.00)	
Logistic	19.291	3.429	0.109		2.175	2.892	1990	3.24*
					(1.24)	(1.00)	(1.00)	
Naslund	1.444	0.195			2.191	2.908	1990	10.20
					(5.00)	(4.20)	(1.00)	
Weibull	19.673	0.039	1.159		2.174	2.900	1993	6.60
					(1.00)	(2.60)	(3.00)	
Tree volume Equation								
Combined variable	0.085	$2.53x10^{-5}$			0.058	0.079	-873	6.84
					(2.44)	(2.18)	(2.22)	
Constant form factor		$2.96x10^{-5}$			0.071	0.096	-720	12.00
					(4.00)	(4.00)	(4.00)	
Logarithmic	$2.22x10^{-4}$	1.434	0.996		0.047	0.069	-978	3.23*
					(1.12)	(1.11)	(1.00)	
Gen. Logarithmic	-0.035ns	$4.25x10^{-4}$	1.331	0.919	0.046	0.068	-977	3.01
					(1.00)	(1.00)	(1.01)	

Note: Values in parenthesis are the relative ranks; ∑R: relative rank sum; * means selected models; ns: not significant.

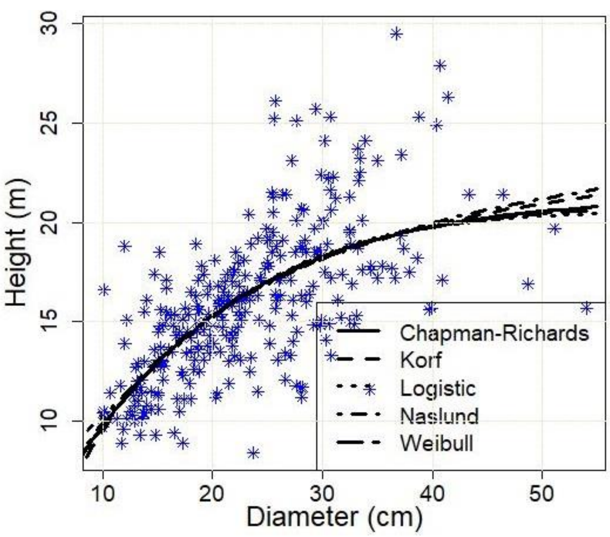

Figure 3. Scatterplot and fitted height trajectories of the five H-D models.

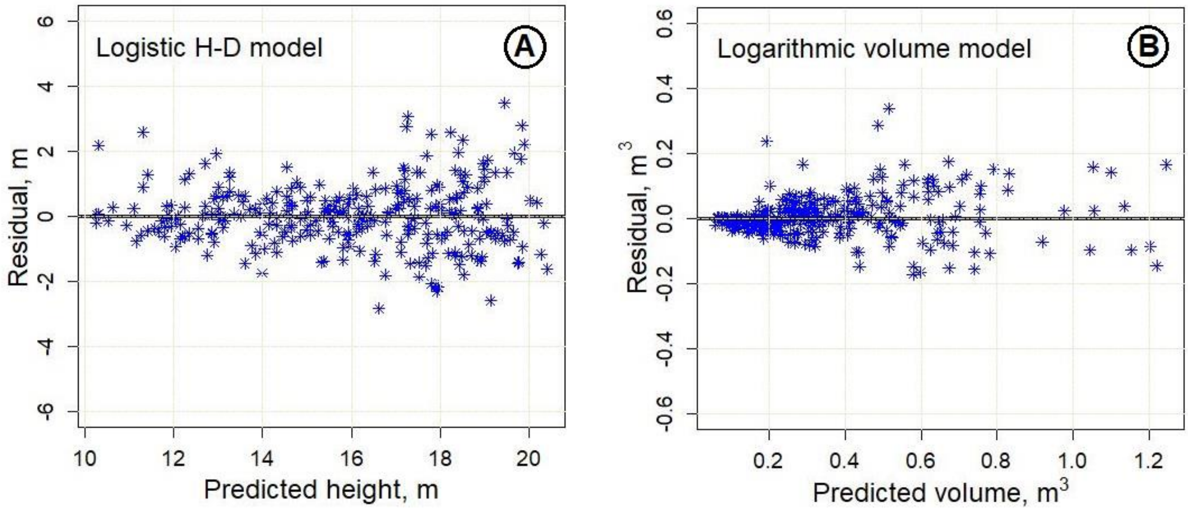

Figure 4. A, Residual of the selected height-diameter model; **B**, Residual of the selected volume model.

The Weibull distribution, Logistic H-D and Logarithmic volume models were used to estimate the yield of the teak stand (Table 5). The table showed the density and yield by 5 cm diameter class. The largest volume of 67.782 m^3 ha^{-1} was observed between 25–30 cm diameter class with a density of 150 trees ha^{-1}. Diameter classes 20–25 cm and 30–35 cm had 59.681 and 57.948 m^3 ha^{-1}, respectively. Thus, majority of the yield of the teak was within 20–35 cm diameter class. The total volume was 309.640 m^3 ha^{-1}.

Table 5. Estimated density and yield of the teak stand from Weibull distribution.

L (cm)	U (cm)	CM (cm)	H (m)	V (m^3)	RF	Density (trees ha^{-1})	Yield (m^3 ha^{-1})	Utilisation purpose
5	10	7.5	9	0.0356	0.00220	2	0.065	Fuelwood (DBH ≤11.0 cm)
10	15	12.5	11.6	0.0954	0.12865	106	10.150	
15	20	17.5	14.1	0.1877	0.22987	190	35.682	Pole (DBH: 12–29.5 cm)
20	25	22.5	16.2	0.3091	0.23347	193	59.681	
25	30	27.5	17.8	0.4527	0.18105	150	67.782	
30	35	32.5	18.8	0.6074	0.11536	95	57.948	Timber (DBH ≥30 cm)
35	40	37.5	19.5	0.7734	0.06235	52	39.879	
40	45	42.5	20	0.949	0.02909	24	22.831	
45	50	47.5	20.2	1.1242	0.01185	10	11.017	
50	55	52.5	20.4	1.3105	0.00425	4	4.606	
Total						**825**	**309.640**	

Note: L = Class lower limit; U = Class upper limit; CM = Class midpoint; H = Class mean height; V = Class mean volume; RF = Relative frequency.

DISCUSSION

This study has evaluated four distributions for describing the structure and estimating the stand volume of the teak plantation in the Oluwa FR. Of the functions evaluated, the Weibull was the most suitable distribution for the teak stand. The beta, Johnson SB and gamma also provided good fits comparable to the observed diameter distribution.

The suitability of the Weibull distribution in predicting diameter distribution in both even-aged and uneven-aged stands are well documented in forestry literature. For example, Sun *et al.* (2019) found the Weibull distribution to be adequate in describing the diameter distribution in the uneven-aged mixed stands of oak and pine. Similarly, Gorgoso *et al.* (2012) also reported good performance with Weibull distribution in characterising the diameter distribution of *Pinus pinaster* Ait, *Pinus radiata* D. Don and *Pinus sylvestris* L. However, some researchers such as Mateus & Tomé (2011), Mayrinck *et al.* (2018), Ogana & Ekpa (2020), etc., reported better fits with the Johnson SB compared to beta and Weibull distributions. Even though the beta and Johnson SB occupy more region in the skewness-kurtosis plane compare to the Weibull (Wang & Rennolls 2005), the Weibull was the preferred distribution for the teak stand based on the evaluation statistics. In addition, it is the only distribution that provides good efficiency in calculating the relative frequency of trees by diameter class due to its closed-form cumulative distribution function (cdf) compares to those of beta, gamma and Johnson SB distributions which require numerical integration.

Modelling the diameter distribution of a forest stand is an integral part of forest management and planning (Nord-Larsen & Cao 2006). This study has provided quantifiable information on product specification for the management of the teak stands. If for example, a diameter size of ≥30 cm is specified for timber, a total yield of 136.281 m^3 ha^{-1} (summation of volumes in classes 30–55 cm) would be obtained from the stand. The product classification was based on Shamaki & Akindele (2014). Other products such as fuelwood, pole, rafter, etc., can be estimated from the system. Effective development of a diameter distribution yield system requires plot data comprising of age, site index (or average dominant height), density, height and diameter for specify sample trees (Burkhart & Tomé 2012). Such that future stand and diameter distribution can be projected for the stands. Though age is a limiting factor in the study at hand, the result can be still be used in the routine management of the teak stand.

CONCLUSION

It is advantageous in forest management to use appropriate statistical distribution in predicting the condition of a forest stand so that silvicultural treatments and harvesting regimes could be prescribed. In this study, we found the Weibull distribution function to be suitable for the teak stand in the Oluwa Forest Reserve. Application of the Weibull distribution model with fitted Logistic height-diameter and Logarithmic volume models provided information on per hectare volume by diameter class for the teak stands. Since the wise use of forest resources is paramount to the forest managers and planners, this information is germane for making decisions on product specification and overall management of the teak stand in Oluwa forest reserve.

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